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TIME DELAY OF VARIATIONAL CURRENT

IN

GLOW DISCHARGE VOLTAGE REGULATOR TUBES

FINAL REPORT--NAVAL RESEARCH PROJECT

CONTRACT NUMBER Nonr-172(00)

TUSKEGEE INSTITUTE

SCHOOL OF ENGINEERING

FINAL REPORT

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GLOW DISCHARGE VOLTAGE REGULATOR TUBES

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NAVAL RESEARCH CONTRACT NUMBER Nonr..172(00)

BY

W. C. Curtis

School of Engineering
Tuskegee Institute
Tuskegee Institute, Alabama

ACKNOWLEDGEMENT

The writer wishes to acknowledge the invaluable assistance rendered by Messrs. Myron R. Ross, Allen H. Turner, and William D. McCullough in making measurements and constructing discharge tubes.

ABSTRACT

If a small signal A.C. voltage is superimposed on the D.C. operating voltage of a gaseous discharge regulator tube, it is observed that there is a large time lag between the flow of the resulting A.C. current and the applied voltage.

The magnitude and phase angle of delay of the small signal A.C. current was determined in function of frequency, D.C. glow current, and voltage by means of bridge measurements. The effects on the variational current of electrode configuration, purity of the gas, and cathode temperature were observed.

It is concluded that the time delay of variational current in reference to applied variational voltage is due principally to the build-up time of ionization. A simple mathematical theory of the time delay mechanism is worked out for certain types of discharges.

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I. INTRODUCTION

A number of papers have appeared in the literature giving measurements indicating that there is a time delay between the flow of the variational current and the application of a small signal variational voltage to a glow discharge voltage regulator tube.[1,2,3,4] When a glow discharge regulator tube is used as a component in an electronic circuit, an accurate theoretical analysis of the circuit must include the effects of the delay current on the action of the circuit.[3,4]

The research described in this report was an investigation of the causes of the time delay of small signal variational current in the voltage regulator tube. The tubes were operated in the normal D.C. glow discharge region, and a small signal alternating current voltage of from 40 to 100 millivolts was superimposed upon the normal D.C. operating voltage. The magnitude and phase delay angle of the resulting small signal alternating current was then determined.

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1 H. Gowchn "On the Non-Stationary Gaseous Discharge," Annalen Der Physic, 20:601, May 1934

2 G. M. Kirkpatrick, "Characteristics of Certain Voltage Regulator Tubes," Proceedings of the Institute of Radio Engineers, 35:487, May 1947

3 Fulvia Iannone and Howard Baller, "Gas Tube Coupling for D.C. Amplifiers," Electronics, 19:106, Oct., 1946

4 W. C. Curtis, "Transient and Low Frequency Response of Electronic Voltage Stabilizers," Harvard University, 1949

The effects of electrode configuration, temperature of the cathode, and purity of the gas on the delay current were observed.

II. PREPARATION OF THE EXPERIMENTAL TUBES

The electrical characteristics of commercial gaseous discharge voltage regulator tubes vary considerably from tube to tube of the same manufacturer except for a very recent design of tube based on the work of Jurriaanse, Penning, and Moubis.[5,6,7] The instability of operation of glow discharge regulator tubes is due to contamination of the cathode by gaseous impurities. The glow itself has a tendency to remove this contamination of the cathode. On the other hand, gaseous impurities coming from the glass walls of the tube tend to again contaminate the cathode. The result of these two opposing actions is to make the glowing spot on the cathode change position in an erratic fashion and to change the starting and operating voltages.

Penning and Moubis [7] point out that a tube containing a carefully cleaned molybdenum cathode charged with a few centimeters of neon gas may be prepared in such a way as to give very stable electrical characteristics.

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5 T. Jurriaanse, F. M. Penning, and J. H. A. Moubis, "The Normal Cathode Fall for Molybdenum and Zirconium in the Rare Gases," Philips' Research Reports, 1:225, April, 1946

6 T. Jurriaanse, "A Voltage Stabilizing Tube for Very Constant Voltage," Philips' Technical Review, 8:272, Sept., 1946

7 F. M. Penning and J. H. A. Moubis, "The Contraction Phenomenon in a Neon Glow Discharge with Molybdenum Cathode," Philips' Research Reports, 1:119, April, 1946

The experimental tubes used in this study were all prepared according to the methods of Penning and Moubis. The cathodes of all of the tubes were constructed of molybdenum. After the cathode was mounted in the envelope, it was very carefully cleaned chemically with a solution of two parts concentrated nitric acid, three parts concentrated sulphuric acid, and one part water. The cleaning solution was then neutralized with a 15% solution of sodium carbonate. The tube was rinsed with tap water, distilled water, and alcohol. It was then dried with a vacuum pump. After the tube was cleaned, it was placed on a vacuum system using an oil pump and liquid air trap. It was pumped to a pressure of at least $1x10^{-6}$mm after a thorough baking in an oven and after the cathode had been heated to incandescence by induction heating apparatus.

The tube while still connected to the liquid air trap, was then filled with commercial spectrographically pure neon to a pressure of 40 mm of mercury. This pressure was measured by an oil manometer. The oil used was Octoil S, and it was degassed by heating it under a vacuum. After filling with neon the tube was sealed off.

When the tube was carefully prepared in this manner, the contraction phenomenon as described by Penning and Moubis took place when a glow discharge of about 10 milliamperes was initiated. As a final action in the preparation of the tube, a current of 100 to 400 milliamperes was passed through the tube for a continuous period of about a week. This caused cathode material to be sputtered on the

walls as an opaque metallic film which protected the cathode from contamination by gases released from the walls. The cathode is at a bright red heat during this process and is, therefore, quite well degassed. Any adsorbed gases on the cathode are removed by the ionic bombardment and by the heat.

Experimental tubes prepared in this manner displayed very stable electrical characteristics when operated in the normal glow discharge region.

III. PHYSICAL APPEARANCE OF THE GLOW

When the glow of the discharge tube was examined, it was found to have certain physical appearances which were different from those considered normal. There was no dark space and the negative glow extended from the cathode for a distance of about one millimeter. The intensity of the light in this region was constant. The light of negative glow dies out slowly in the Faraday dark space and has a red color in this region indicating that the gas is quite pure and that metastable neon atoms are present.[8]

When the light of the negative glow is viewed through a spectroscope, all of the important arc lines are visible and no impurity lines can be seen. A measuring microscope with a resolving power of .001 millimeters was used in an attempt to observe the dark space of the discharge but none could be found.

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8 M. J. Druyvesteyn and F. M. Penning, "Electrical Discharges in Gases," Reviews of Modern Physics, 12:130, April 1940

IV. VARIATIONAL CURRENT AND PHASE CHARACTERISTICS

Measurements of the delay characteristics of the glow discharge tubes were made on a conventional Hay Bridge so arranged that the direct current and voltage necessary for the operation of the tube in the normal glow discharge region could be supplied. The equivalent conductance and susceptance of the glow discharge were measured, and the nagnitude and phase angle of delay of the variational current with reference to the variational voltage were computed. Figures 1, 2, and 3 show the variation of the magnitude and phase angle of delay of the variational current in reference to the variational voltage across the glow tube. The small signal voltage across the tube for these measurements was maintained at 40 millivolts.

The experimental tube used for these measurements had plane parallel electrodes 2 cm square spaced 6 millimeters apart. There was no positive column or anode glow in the discharge space. A direct current of 30 milliamperes completely covered the inner cathode surface with glow. A stable discharge current as low as two milliamperes could be maintained. The glass envelope was about 15 centimeters in diameter. This dimension was made large so that the envelope would not affect the characteristics of the discharge. The tube was filled with neon to a pressure of 30.3 millimeters of mercury.

From Figures 1 and 2 it will be noticed that at high frequencies the variational currents decrease with a decrease in the value of the D.C. glow current. At low frequencies this is not the case. In Figure 1,

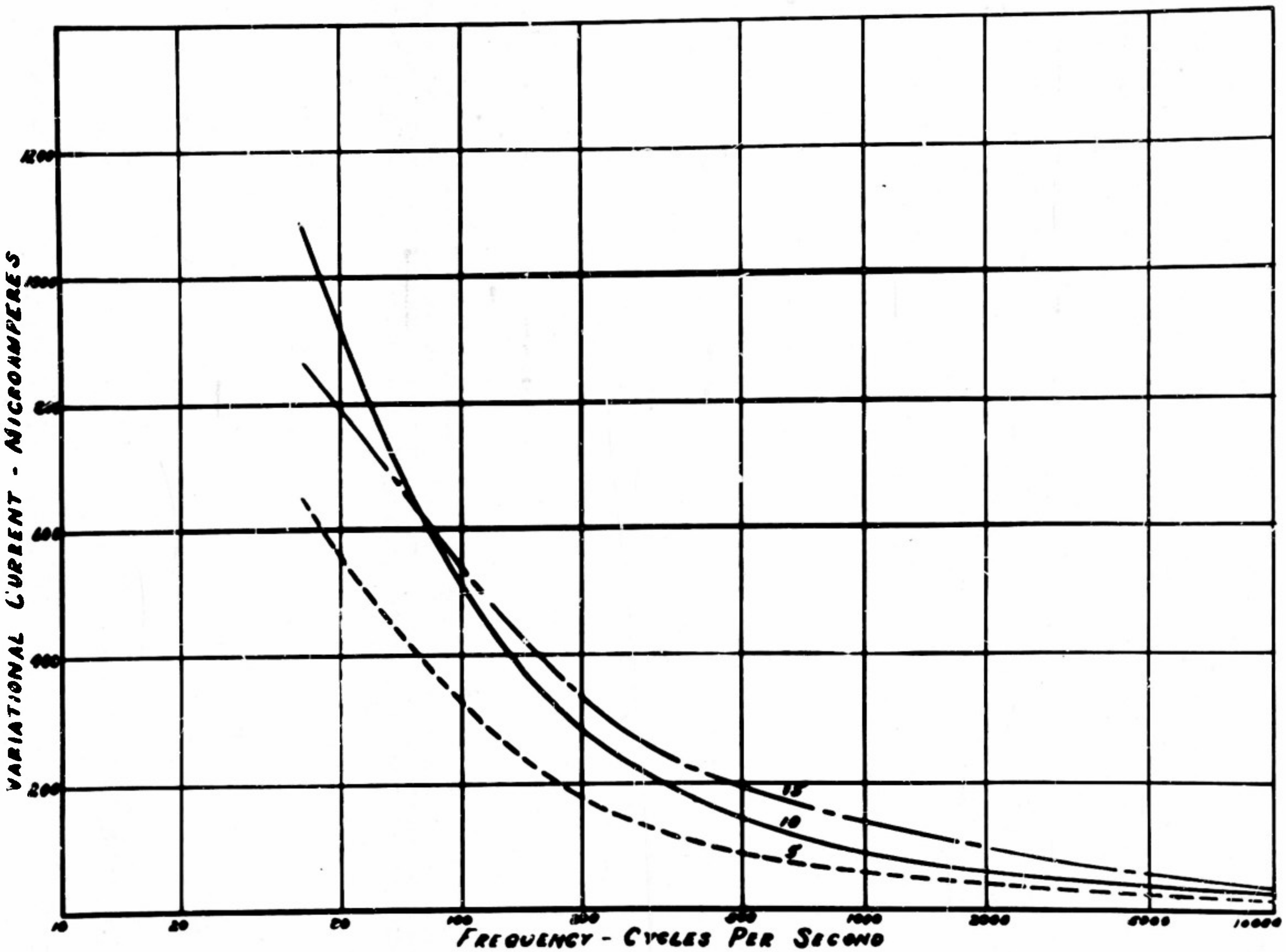

Fig. 1. The magnitude of the small signal current in function of the frequency of the variational voltage. The magnitude of the voltage for all measurements was 40 millivolts. Parameter-direct glow current in milliamperes.

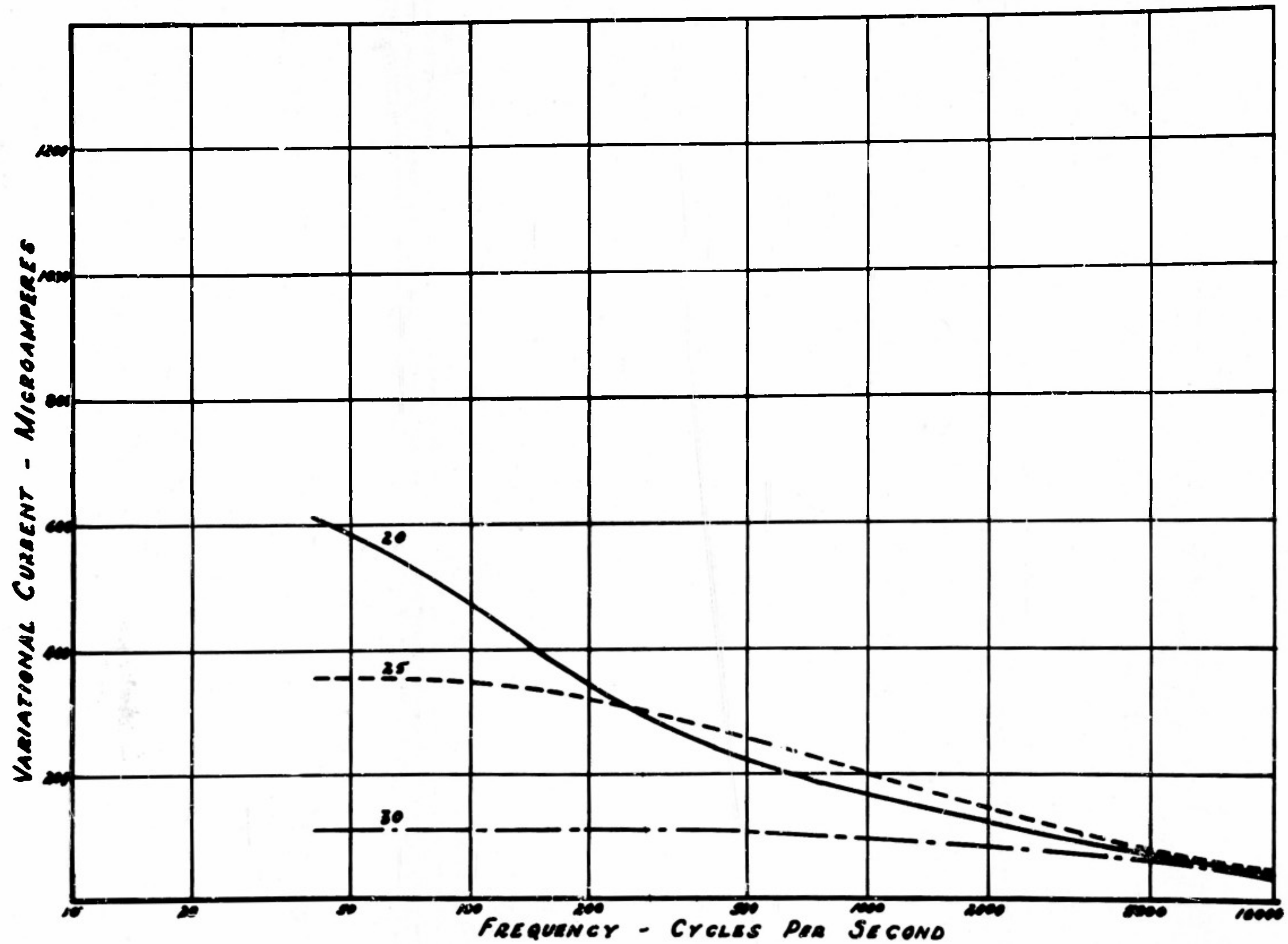

Fig. 2. The magnitude of the small signal current in function of the frequency of the variational voltage. Magnitude of the voltage - 40 millivolts. Parameter direct glow current in milliamperes.

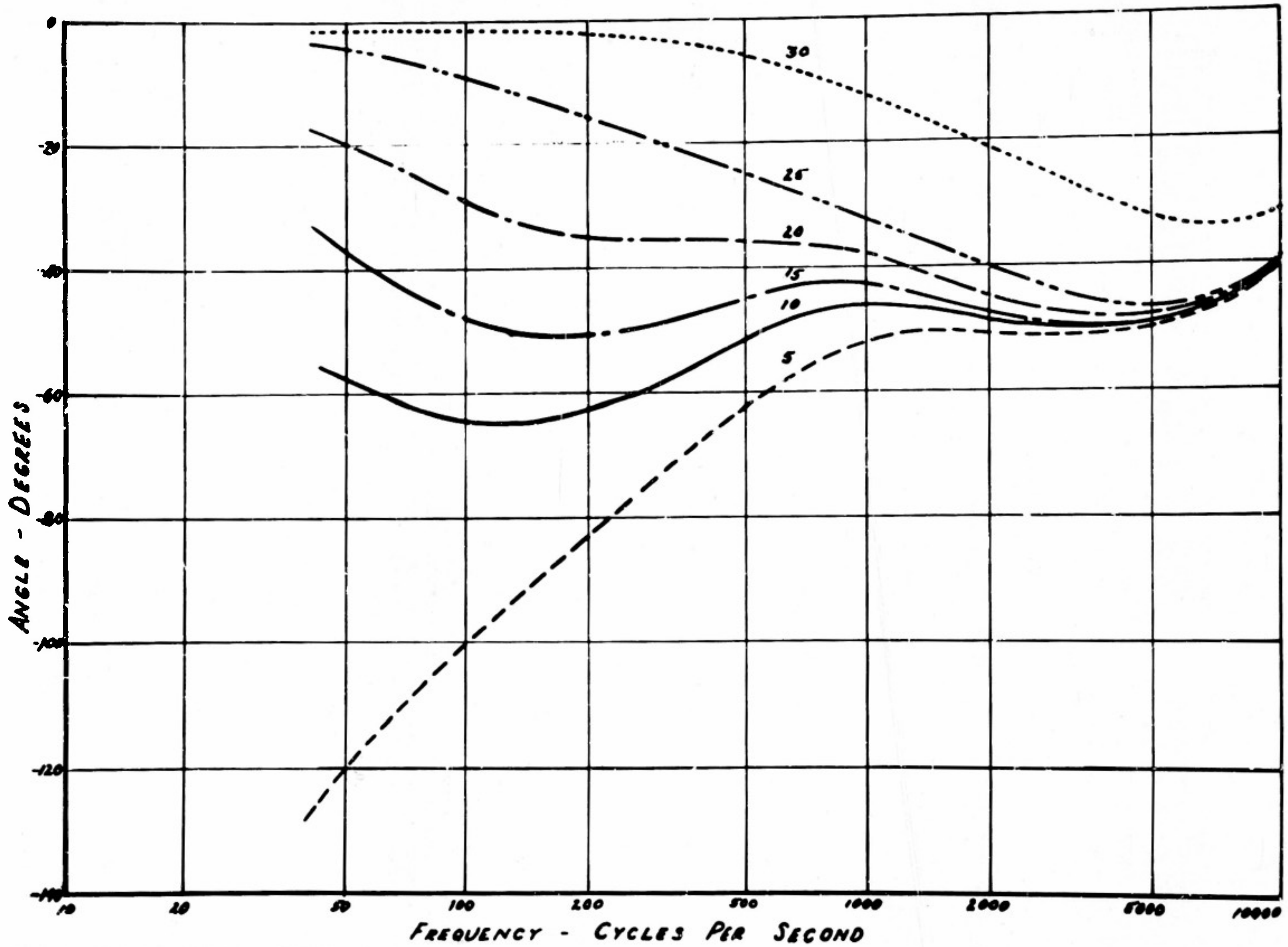

Fig. 3. Phase angle of the small signal current in function of the frequency of the variational voltage. Parameter direct glow current in milliamperes.

the variational current for 15 milliamperes D.C. is less than that for 10 milliamperes D.C. The size of the glow spot at 15 milliamperes D.C. is of course larger than that at 10 milliamperes. This indicates that some factor other than small perturbations in the ionization is producing the current.

An examination of the phase angle of delay of the small signal current in relation to the applied voltage shows that the angle decreases with frequency for high glow currents and increases with decreases in frequency for the lowest current. It would seem that the mechanism of the low frequency time delay at low glow currents is not the one effective at high currents. At high frequencies, the delay angle is about the same for all D.C. glow currents which indicates that here the same mechanism of delay is operating in all cases. It is clear, also, that the mechanism involved is not based on a constant time delay except for possibly the highest glow currents.

V. EFFECT OF ELECTRODE CONFIGURATION ON VARIATIONAL CHARACTERISTICS

In section IV, the variational current characteristics of a glow discharge tube with plane parallel electrodes was discussed. It was thought that there was a possibility that the electrode shape might have some effect on the mechanism.

If the anode is a small wire located in the tube axially with respect to a plane circular cathode, the variational current characteristic differs little from those for the plane parallel case provided that the current density is not so high as to cause an anode glow. At the currents of a

normal glow discharge, this condition is easily met.

Referring to Figure 3, it will be noted that as the direct current of the glow is decreased from 30 to 5 milliamperes, the phase angle of delay of the variational current increases to quite large values. Two physical factors related to the glow spot size should be noted. One of these is the fact that as the D.C. glow current decreases, the spot size decreases in direct proportion. If 30 milliamperes of glow current completely covers the inner surface of the cathode, then a 15 milliampere glow current would cover only half of the cathode surface. This means also, that at this glow current, one-half of the cathode surface is not covered by glow. If metastable atoms of neon escaping from the negative glow which possibly could not survive a transit to the cathode surface in the area directly under the spot are now able to strike the uncovered parts of the cathode, electrons would be ejected from the cathode by these atoms. There would be a considerable time delay in the arrival of these metastable atoms at the cathode since they travel by thermal diffusion. This could possibly be the cause of the time delay in the tube at low frequencies.

The other physical factor related to glow spot size is connected with the idea that the glow spot size is essentially circular. As the glow current decreases, the cross-sectional area of the spot decreases as the radius squared while the circumference decreases directly with the radius. If the loss of charged particles through the circumference of the glow spot is important to the efficiency of the ionization process

in the glow, then it would be reasonable to expect the build-up time of small perturbations in ionization caused by the variational voltage to be longer as the spot size decreased. This might be a possible cause of the delay.

As a check on the second of these factors, a tube was designed in which the edge of the spot, through which ions from the discharge could escape without taking part in the discharge, would be of constant length as the spot increased in size. This was accomplished by using a cathode of 2 millimeter molybdenum tubing about 8 centimeters long. The anode was a 0.6 millimeter wire running parallel with the cathode and separated from it by 5 millimeters. This tube was prepared in the usual manner and was thoroughly sputtered after the contraction phenomenon took place.

The discharge always began at one end of the cathode and spread along it as the D.C. glow current was increased. Twelve milliamperes of glow current completely covered the cathode. The gas pressure of neon in the tube was 37.4 millimeters of mercury. The length of the edge of the discharge was always approximately equal to the periphery of the cathode. The glow then always had a constant length of edge, through which charged particles from the discharge could be lost, regardless of the size of the glow spot.

Figures 4 and 5 give the results of measurements of this tube. It will be noted that neither the amplitude nor phase of the variational current varies as greatly as in the case of the tube with plane electrodes. The shape of the phase angle curves in Figures 3 and 5 indicates that the

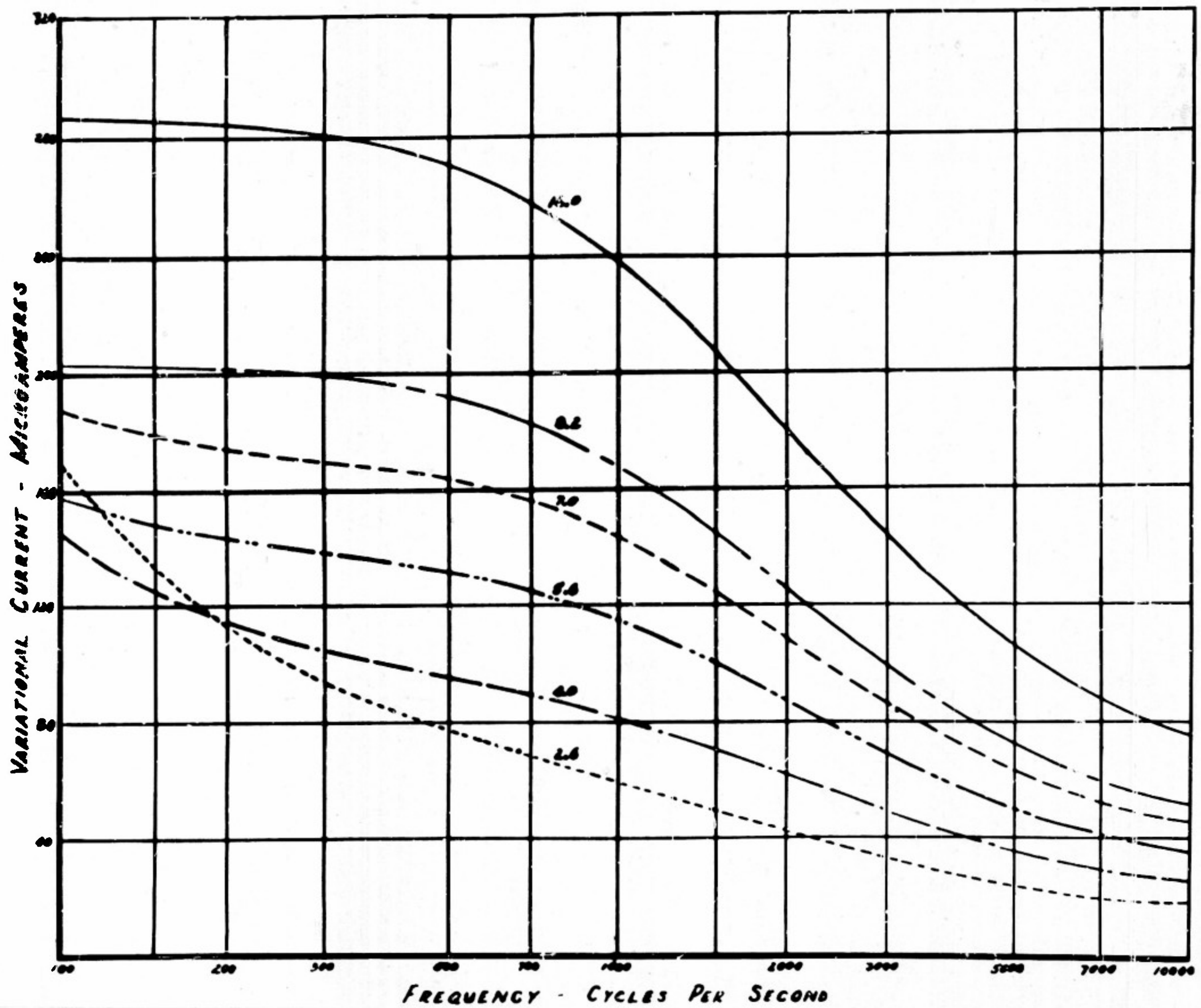

Fig. 4. The variational current, in function of frequency with D.C. glow current as a parameter, of a glow tube with a cylindrical cathode.

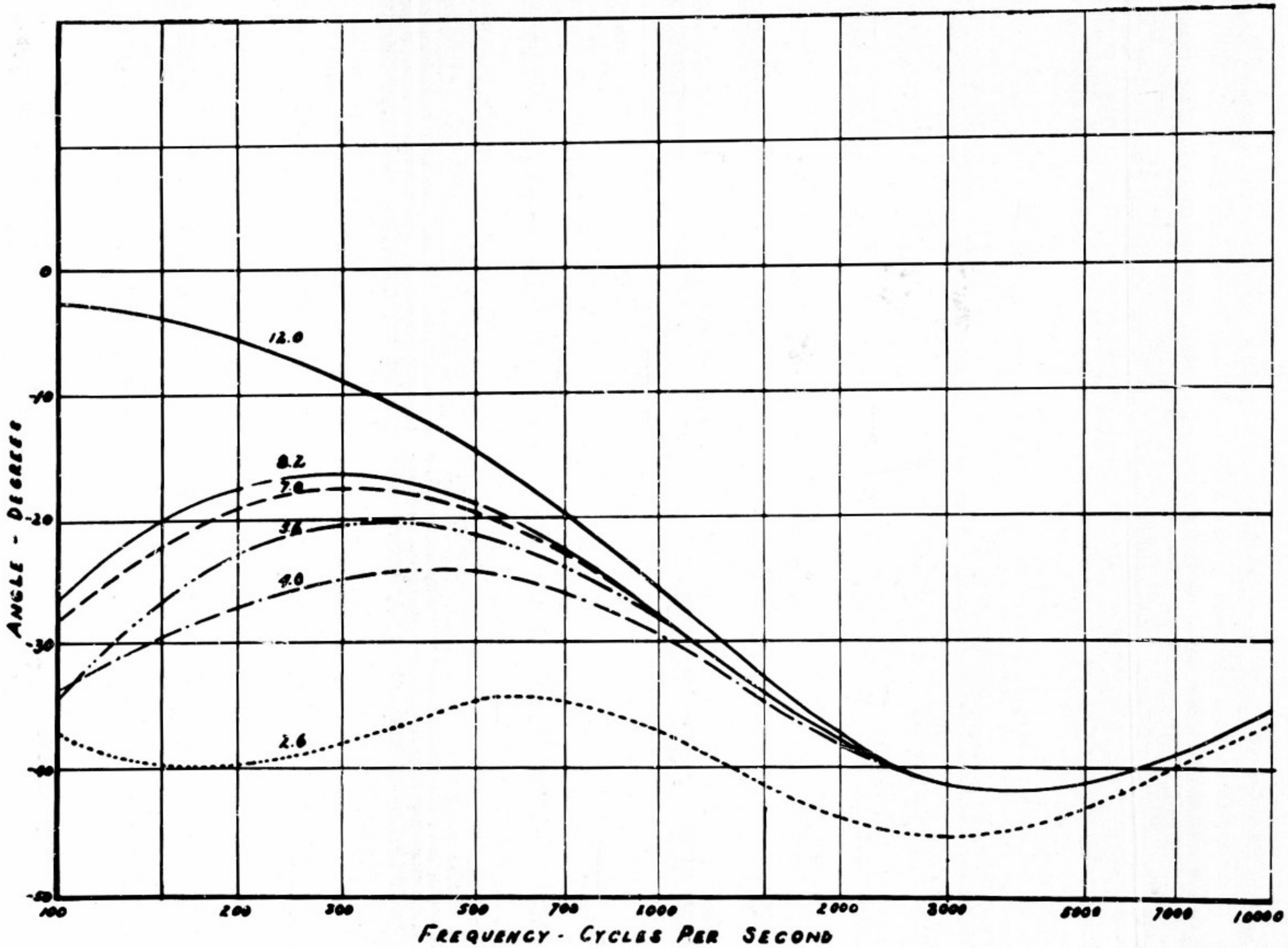

Fig. 5. Phase angle of delay of the variational current, in function of frequency with D.C. glow current as a parameter, of a glow tube with a cylindrical cathode.

same mechanism is active in both cases at frequencies above 5,000 cycles and that a mechanism active at about 1,000 cycles in the plane parallel case is not effective in the cylindrical cathode case until about 300 cycles. It would seem that at 2.6 milliamperes the spot displays characteristics in the cylindrical case which are similar to the case of the plane parallel electrodes at 10 and 20 milliamperes.

VI. EFFECT OF ARGON IMPURITY

It is well known that metastable neon atoms of a glow discharge are very efficiently destroyed by introduction of a few tenths of a per cent of argon. This is due to the fact that the ionization of argon is 15.69 volts while the two metastable states of neon are 16.67 and 16.76 electron volts. If metastable atoms are striking the cathode in appreciable numbers and are producing delayed currents as a result of the fact that they travel to the cathode only by thermal diffusion, it should be possible to reduce the delayed current considerably by the introduction of small amounts of argon.

To examine this idea experimentally a tube with plane parallel electrodes two centimeters square and separated by 6 millimeters was used. A side tube containing a breakable capsule of argon was attached to the experimental tube. The side tube was provided with a barium getter, which was flashed after the main tube was charged, to prevent any gaseous impurities evolved by the walls of the side tube from entering the main discharge tube. The contraction phenomenon was obtained in the main tube and the walls were thoroughly sputtered. A series of measurements were

made of the delay characteristics of the tube for the pure neon discharge for various values of glow current. Characteristic curves such as those of Figures 1, 2, and 3 resulted.

The argon capsule in the side tube was then broken, and the nature of the discharge changed radically. The tube was being operated at ten milliamperes of glow current. When the argon was introduced the glow current increased to about 12 milliamperes. The glowing spot which covered about one-third of the inner surface of the cathode spread over both the back and front of the cathode. The D.C. operating voltage dropped from 110 to about 91 volts which is characteristic of a neon discharge with a small impurity of argon.

The argon capsule contained enough gas to produce a 0.4 per cent impurity. A new series of measurements were then made on the discharge tube to determine the time delay of the variational current.

Although the introduction of the argon produced spectacular changes in the physical appearance of the discharge, it made little difference in variational characteristics. Figures 6 and 7 give the results of the introduction of the 0.4 per cent argon impurity. Note that at low frequencies the argon causes a decrease in the amplitude and a decrease in the phase angle. Both of these, however, are small. In the midband of frequencies the variational amplitude increases with the introduction of the impurity. At high frequencies there is no effect on the amplitude. There is, however, a greater phase delay at the high frequencies.

If metastable neon atoms are playing a major role in the production

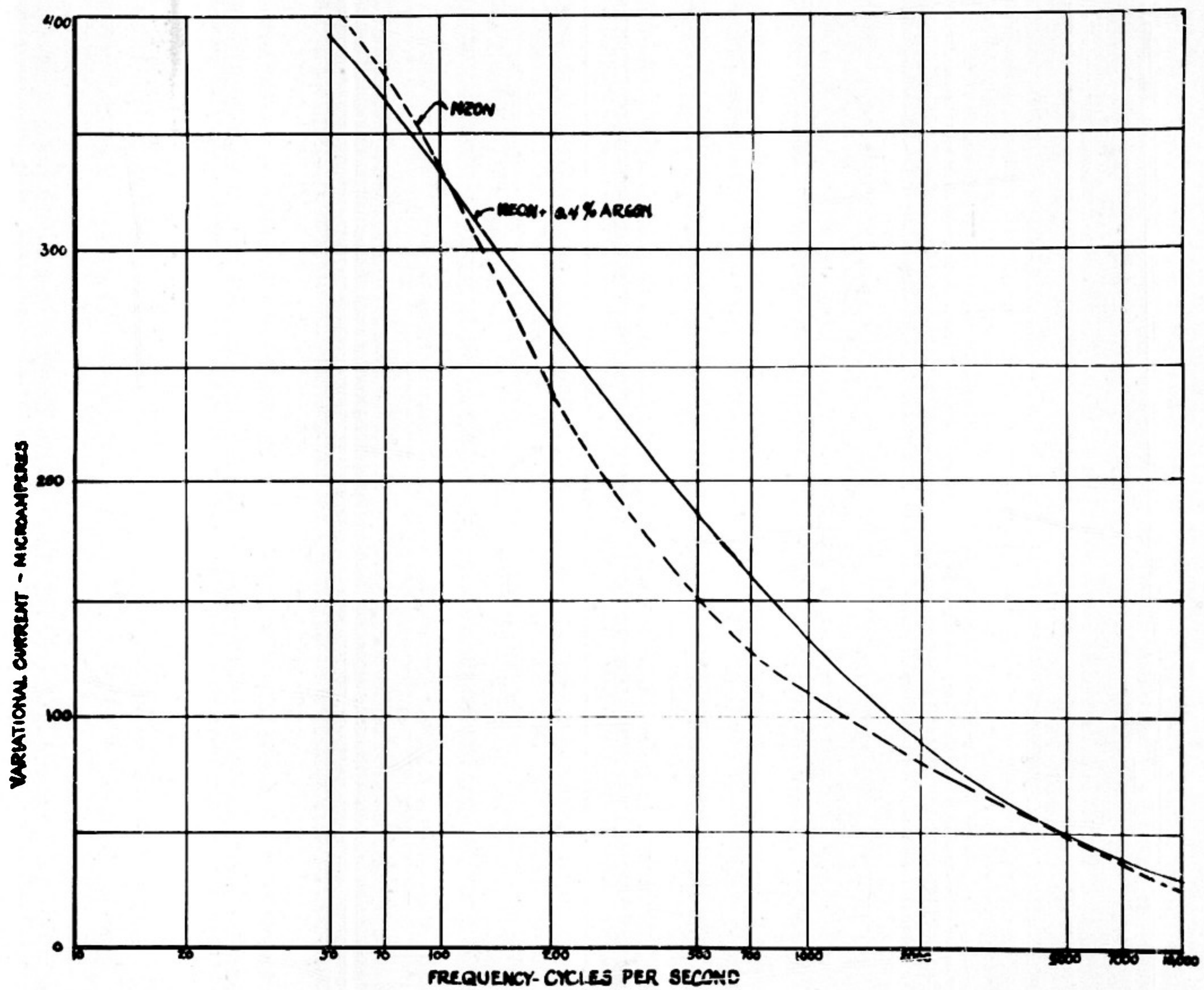

Fig. 6. The effect of a 0.4% argon impurity on the variational current of a neon glow discharge with plane parallel electrodes.

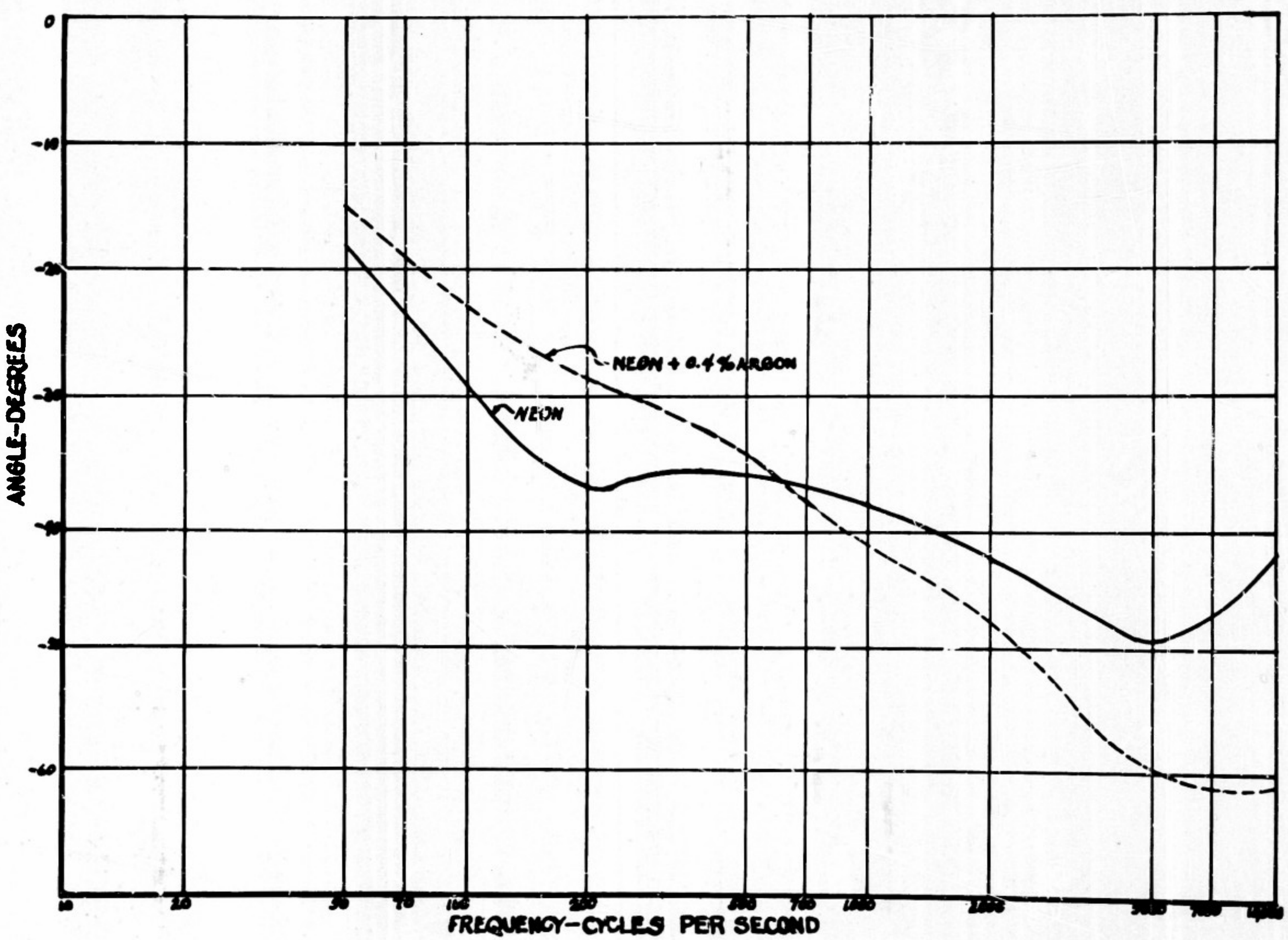

Fig. 7. The effect of a 0.4% argon impurity on the variational phase characteristic of a neon glow discharge.

of the variational current, it would be expected that this current would decrease very radically at about 100 cycles. There is only a 5 per cent decrease. The phase angle decreases from 29 to 24 degrees.

A metastable atom striking the cathode will almost always produce an electron which will go on to produce an avalanche. On the other hand, a metastable atom ionizing an argon atom produces an electron but since it is not necessarily at the cathode, it will not produce an electron avalanche. The positive ion so produced will have a chance of about one in twenty of producing an electron when it strikes the cathode. Therefore, it is clear that since the introduction of argon does not change the variational current a great deal, the metastable atoms must not be carrying out a major role in the production of the variational current.

VII. EFFECT OF CHARGES ON THE WALLS

Some charge carriers of any gaseous discharge move to the walls and are lost. Very often metastables have lifetimes sufficient to reach the walls and sometimes are produced close to the walls by radiation from the discharge. Since it was possible that the walls may have played a part in the production of the delayed variational current, a tube was constructed with an envelope of 30 millimeter glass tubing and a cathode 24 millimeters in diameter was mounted in the envelope perpendicular to the axis of the envelope and 3 millimeters from the walls. There was a one centimeter-six-tenths millimeter diameter wire along the axis of the tube as an anode. A small wire was also sealed into the wall of the

envelope in process of preparation of the tube. Here again in the preparation of this tube the contraction phenomenon was obtained.

In this tube it was found that there was a D.C. voltage of the walls with respect to the cathode such that the net current to the walls was zero. This was to be expected. In the tube tested, it was about 48.5 volts. If the potential of the walls was changed by as much as 10 volts plus or minus from this value, there was no noticeable change in the value of the variational current. This led to the conclusion that the walls, even when close to the discharge, had no important effect in the variational characteristic of the tube.

VIII. EFFECT OF TEMPERATURE OF THE CATHODE

In the process of periodically sputtering the tubes at glow currents of about 400 milliamperes, the cathode is heated to a cherry red color. If the glow current is suddenly reduced to a normal value of say 5 milliamperes, it is observed that the glow covers the entire cathode. The glow contracts as the electrode cools, and returns to its normal size for a 5 milliampere current in about 5 minutes. This suggests that the temperature of the cathode may affect the amplitude of variational current and the phase angle.

To investigate this phenomenon, a tube was constructed with a helical cathode of 0.7 millimeter molybdenum wire. The diameter of the helix was 10 millimeters and a wire anode was located at its axis. The turns of the helix were very close together and a circular spot covering about one-fourth of the surface formed on the inside of it when a 5 milliampere

glow current was passed. Both ends of the helix were brought through a press, and it was possible to pass a current through the helix. A D.C. current of 10 amperes caused the helix to have a dull red color and 15 milliamperes made it incandescent. If the 5 milliampere glow current was held constant and heating current started, it was found that the glow spot completely covered the helical cathode when its temperature approached incandescence. If it is assumed that the energy of the discharge is principally dissipated at the cathode, it would be possible to compute this energy and determine the values of D.C. helix current which would produce the same energy at the cathode. Figure 8 is a plot of the relation of variational current to D.C. glow current in the curve marked with circles. The curve marked with crosses shows the effect on the variational current of heating the helix with filament current. In this case the glow current was held at 5 milliamperes. The breaks in the curve are due to the fact that the cathode is composed of a closely wound helix which did not act as a perfectly smooth cathode. The glowing spot on occasion jumps to a new position as the heating or glow current varies because the cathode is not smooth.

Figure 9 shows the relation of variational voltage to the temperature of a liquid-cooled cathode. The experimental tube is that described on Page 8 of this report. The cathode consisted of a molybdenum tube through which a cooling liquid could be passed. The glow current was 4 milliamperes, the variational voltage 100 millivolts, and the frequency was 500 cycles for these measurements.

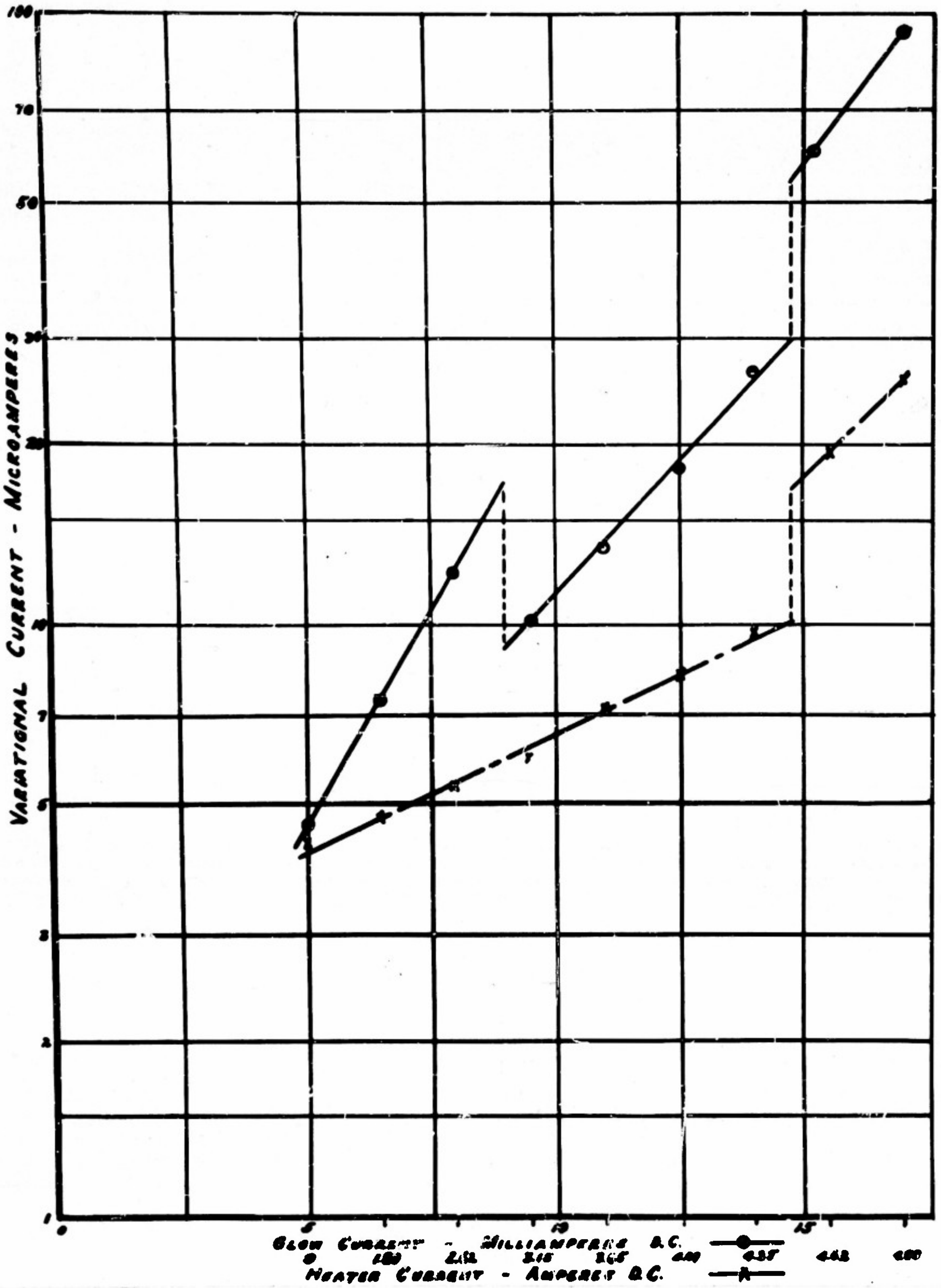

Fig. 8. The relation of the variational current to cathode heating. Frequency 1,000 . Variational voltage 40 millivolts.

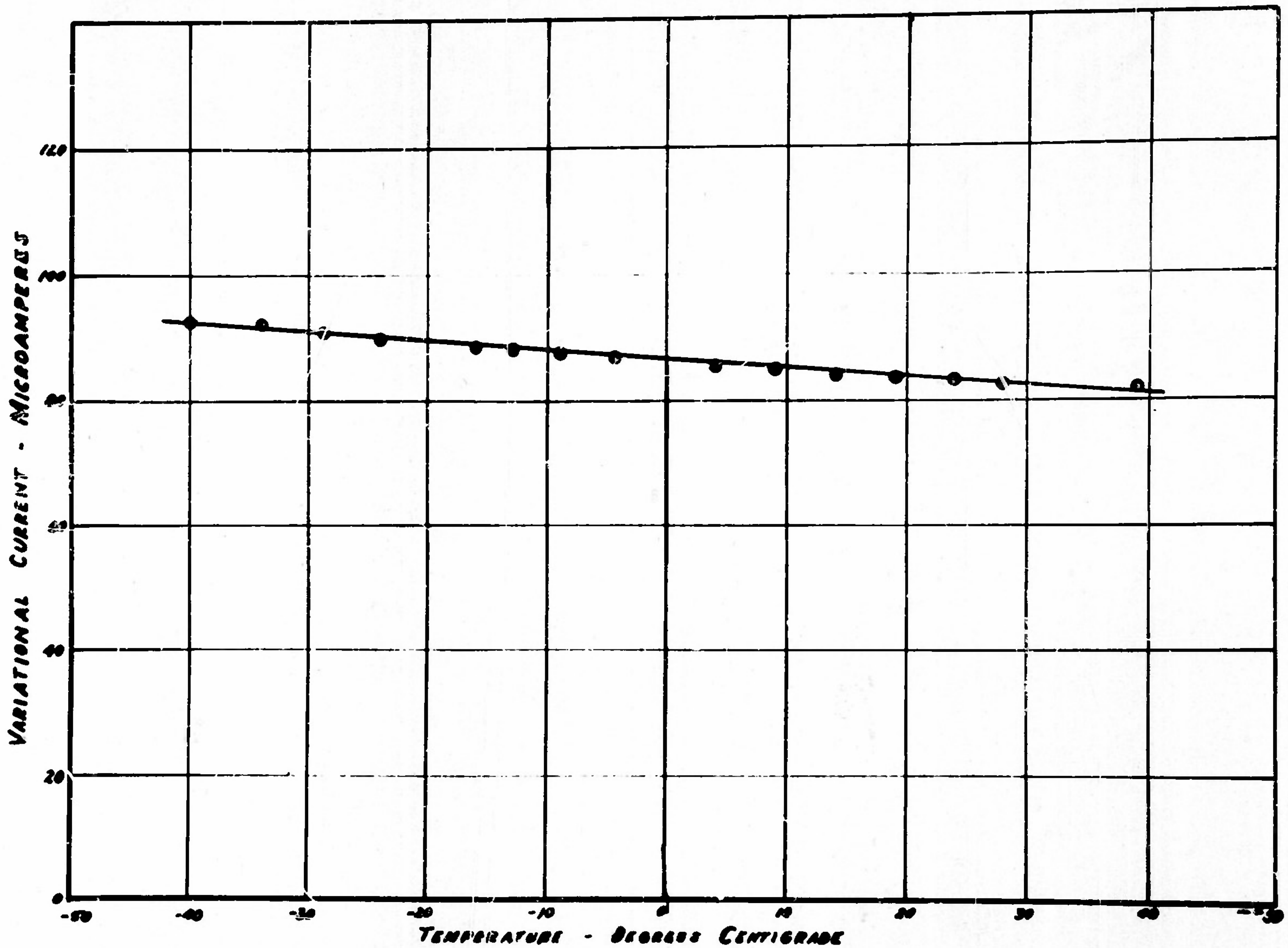

Fig. 9. Relation of variational current to temperature of the cathode. Glow current 4 milliamperes, variational voltage 100 millivolts, frequency 500 ~.

It is clear from both of these measurements that the variational current is linearly related to the temperature of the cathode. Because of the very small changes in variational current for the sort of variations in temperature of the cathode expected under normal operation, it was felt that temperature variations at the cathode could not be the major factor in the production of the time delay of the variational current.

Mention should be made of the fact that the investigators were conscious of the fact that the data of Figure 7 was influenced by the small voltage drop of some few tenths of a volt which existed along the cathode due to the glow of filament current. This voltage drop did have some effect on the variational current as could be verified by reversing the heating battery connections to the helix. However, the effect was small.

IX. SIMPLE THEORY OF THE VARIATIONAL CURRENT OF THE GLOW DISCHARGE AT HIGH CURRENTS

An examination of the curves of Figures 2, 3, 4, and 5 shows that the amplitude and phase relation of the variational current to frequency is simpler for high glow currents than in the cases for low glow current. This is particularly true for the delay angle. The highest glow currents were those which just covered the cathode with glow. For the lower values of glow current in these measurements only a small portion of the cathode surface was covered with the glow.

If it is postulated that the large changes in variational current at low frequencies as compared to those at high frequencies is due to the fact that at low frequencies the spot has time to expand and contract

with the variational changes in voltage, this would account for the large rise in variational current at low frequencies and low glow currents. At high currents the spot cannot spread because the cathode is already covered. At the edge of the glow spot for low currents, the discharge is barely able to maintain itself and will therefore increase and decrease in size for very small changes in the voltage of the discharge. This is why the D.C. discharge voltage remains essentially constant over large variations in glow current. On the other hand, if the glow completely covers the cathode, the spot is not able to expand to the point of delicate balance mentioned for low currents. Therefore, small perturbations of the voltage at low frequencies will not produce large changes in variational current when the cathode is covered by glow.

The variational current at low frequencies and high current must be produced by the same meahcnism which produces this variational current at high frequencies for all currents. Note that at high frequencies the amplitude and phase of all variational currents merge in Figures 1, 2, 3, 4, and 5 which indicates that the same mechanism is operative at these frequencies. The glow discharge which cannot change size with variational voltage will be referred to as a constrained discharge. The process of expansion and contraction of the glow is slow. When the discharge is viewed through a synchronized shutter, the spot can be seen to expand and contract with variational voltage at low frequency but steadily decreases in amplitude with increases in frequency and ceases at about 700 cycles. A discharge is therefore constrained if the frequency is

above about 700 cycles. The discharge is also constrained if the glow completely covers the cathode. Here the glow cannot expand and contract with variational voltage because the spot cannot expand under the influence of the D.C. operating voltage to the point where the discharge at the edge of the glow can barely maintain itself.

A simple theory has been worked out for the constrained discharge. It is assumed that as a result of the superimposition of a small A.C. voltage on the D.C. operating voltage of the glow discharge, that positive ions are released from the potential maximum of negative glow in much the same manner that electrons are released from the potential minimum in a vacuum tube by a change in plate voltage. These positive ions travel to the cathode and release electrons which in turn produce more positive ions which again travel to the cathode causing the release of more electrons and continuing the process. It is further assumed that some average transit time τ relates the time of arrival of succeeding groups of positive ions and that the magnitude of each group is related to the preceding group by the factor $e^{-\alpha\tau}$.

Under these conditions the current of ions released at the negative glow can be described as $I_o e^{j\omega t}$. These ions release electrons at the cathode which produce a positive ion current $I_o e^{j\omega t} e^{-\alpha\tau} e^{-\omega\tau}$. The factor $e^{-\alpha\tau}$ represents the dying out of the disturbance. The factor $e^{-\omega\tau}$ represents the time delay between pulses of ion current at the cathode. The total current at the cathode is

$$i = I_o\left\{e^{j\omega t} + e^{j\omega(t-\tau)}e^{-\alpha\tau} + e^{j\omega(t-2\tau)}e^{-2\alpha\tau} + \cdots\right\} \quad (1)$$

$$i = I_o e^{j\omega t}\left\{1 + e^{-(\alpha+j\omega)\tau} + e^{-2(\alpha+j\omega\tau)} + \dots\right\} \tag{2}$$

The ratio of terms $\frac{n+1}{n} = e^{-(\alpha+j\omega)\tau} = K$

$$i = I_o e^{j\omega t}\left\{1 + K + K^2 + K^3 + \dots\dots\right\} \tag{3}$$

$$i = \lim_{n\to\infty} I_o e^{j\omega t}\left\{\frac{1-K^n}{1-K}\right\} = \frac{I_o e^{j\omega t}}{1-K} = \frac{I_o e^{j\omega t}}{1-e^{-(\alpha+j\omega)\tau}} \tag{4}$$

We define $e^{-\alpha\tau}$ as p and take the real part of (4)

$$i = \mathcal{R}\left[\frac{I_o(\cos\omega t + j\sin\omega t)}{1 - p\cos\omega\tau + j p\sin\omega\tau}\right]$$

$$i = \frac{I_o}{\sqrt{p^2 - 2p\cos\omega\tau + 1}}\cos(\omega t - \phi) \tag{5}$$

where

$$\phi = \tan^{-1} \frac{p\sin\omega\tau}{1 - p\cos\omega\tau} \tag{6}$$

The real part of the current is

$$I_R = \frac{I_o(1-p\cos\omega\tau)}{p^2-2p\cos\omega\tau+1} \tag{7}$$

The imaginary part of the current is

$$I_L = \frac{I_o p \sin\omega\tau}{p^2-2p\cos\omega\tau+1} \tag{8}$$

Most measurements show a maximum value of I_L. To compute this we take

$$\frac{dI_L}{d\omega} = 0$$

from which we find

$$\cos\omega\tau = \frac{2p}{p^2+1} \tag{9}$$

and

$$\sin\omega\tau = \frac{p^2-1}{p^2+1}, \frac{1-p^2}{p^2+1} \tag{10}$$

The second of these roots is physically realizable. At the point where I_L is maximum

$$\frac{I_R}{I_L} = \frac{1-p\cos\omega\tau}{p\sin\omega\tau} = \frac{1}{p} = \frac{1}{e^{-\alpha\tau}} \tag{11}$$

That is, the ratio of the real to the quadrature components of current is equal to the ratio of the amplitude of successive pulses of positive

ions reaching the cathode.

The factor τ can also be calculated by the equation

$$\tau = \frac{1}{\omega} \cos^{-1} \frac{2p}{p^2+1} \qquad (12)$$

The measured curves of amplitude and phase delay angle for 12 milliamperes in Figures 4 and 5 will be used to check the theory.

Figure 10 shows the measured values of the real and quadrature components of variational current in the glow discharge tube. The cathode was fully covered at 12 milliamperes, and the discharge was therefore constrained. Note that the maximum value of I_L is at 1500 cycles per second. The ratio of I_R to I_L yields the value of p equal to 0.645 and a value of τ equal to 4.45×10^{-5} seconds. The equation was fitted to the curve by computing I_o from the equation for I_L at 1500 cycles.

Figures 10, 11, and 12 show the relation of the theoretical curves to the measured curves for 12 milliamperes. The fit at frequencies below 1500 cycles for the amplitude and phase angle is excellent. The amplitude curve is good to 10,000 cycles as shown in Figure 11 while the calculated phase angle curve is a very poor fit above 4,000 cycles in Figure 12.

The value of p = 0.645 which is the ratio of the amplitudes of successive bunches of ions arriving at the cathode, and the value of τ which is the average transit time of the ions, indicate that ions due to a variational disturbance die out to one per cent of their initial value after about ten transits. There is no data available to check the

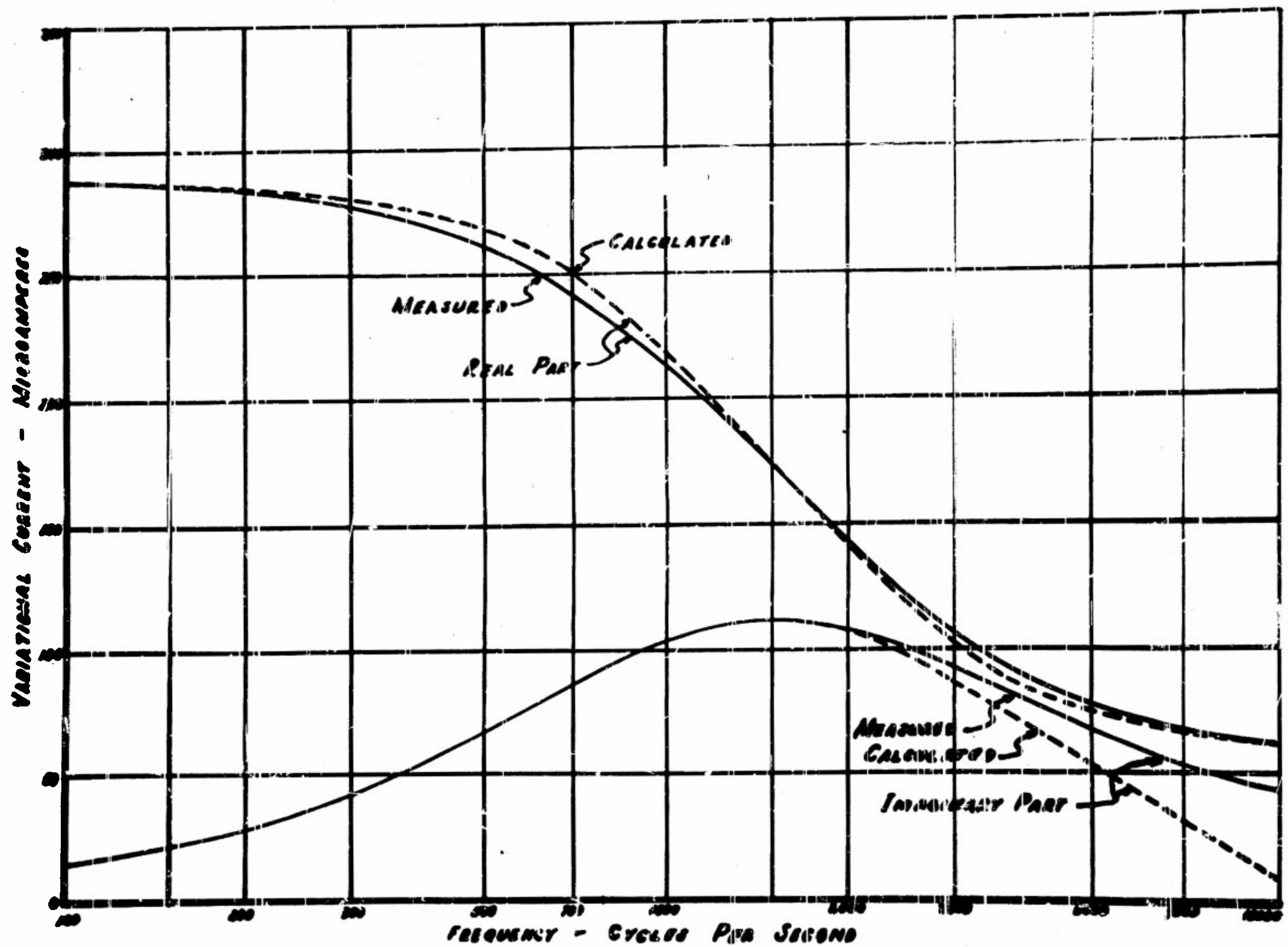

Fig. 10. Calculated and measured values of the real and imaginary parts of the variational current for a constrained discharge.

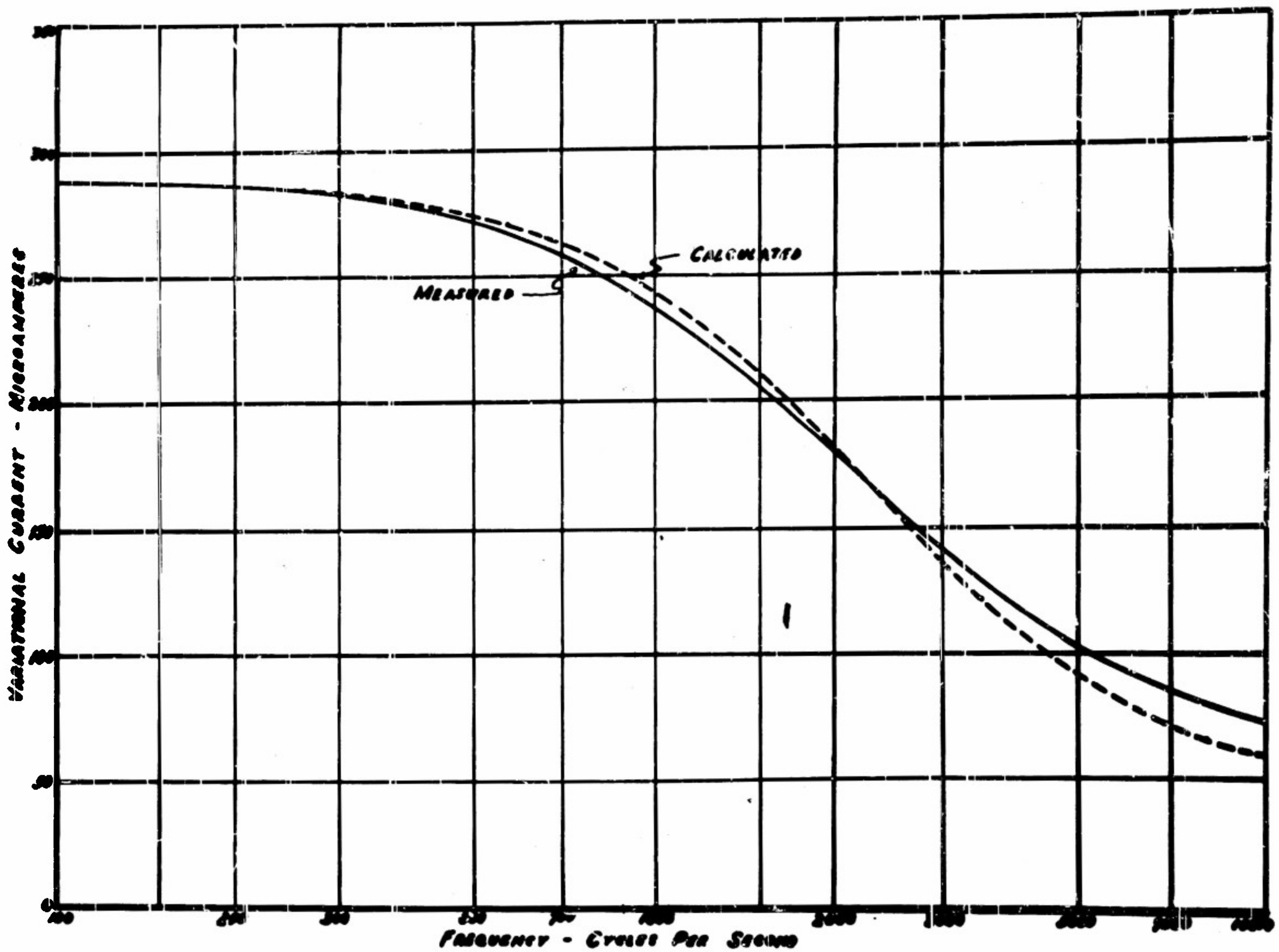

Fig. 11. Amplitude of the calculated and measured values of variational current for a constrained discharge.

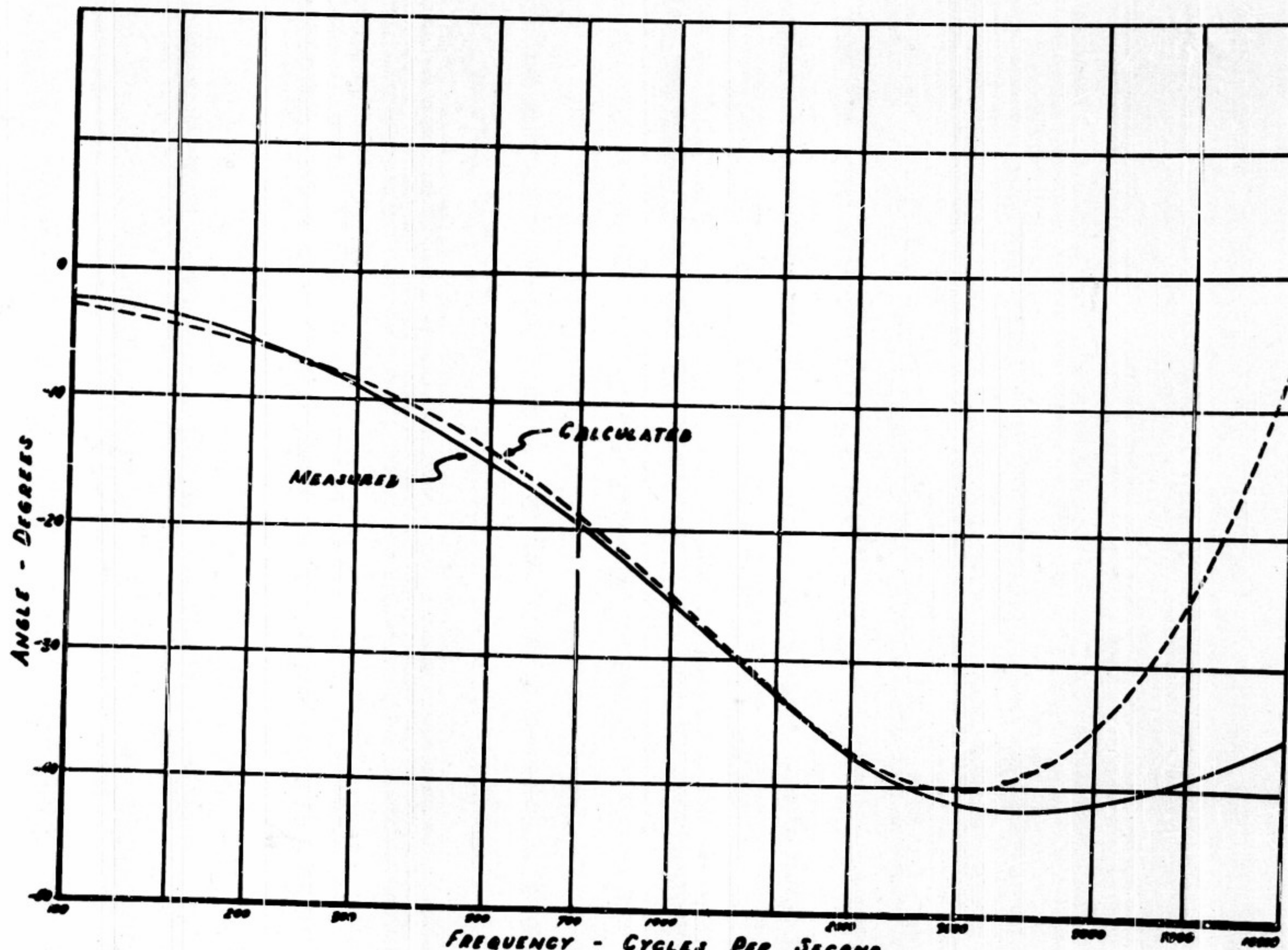

Fig. 12. Phase angle of delay for the constrained discharge.

transit time.

One of the failures of this theory is that it predicts a periodicity in the amplitude of the variational current in function of frequency. From the constants of the curves of Figures 10, 11, and 12 the theory shows that the amplitude of the variational current should be a maximum at zero, and integral multiples of 22,500 cycles. These peaks in amplitude have not been detected in a number of attempts. This failure of the theory is probably due to the fact that positive ions are produced throughout the dark space and will not have a common transit time. A mathematical theory following the work of Kruithof [9] will give a solution which does not show the periodicity. The solution does not give as good a fit to the measured curves.

X. VARIATIONAL CURRENT OF LOW GLOW CURRENTS AND LOW FREQUENCY

At low glow currents the stability of the glow on the cathode depends on a very critical balance. The spot spreads until at its edge, the discharge is barely able to maintain itself. As a result of this very delicate balance between charged particle losses and production, any change in the voltage of the glow such as the superposition of a small A.C. voltage will make relatively large changes in the current. Since there is a balance between losses and production of charged particles at the edges of the discharge, one would expect the spread or decay of the glow

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9 A. A. Kruithof, "Time Lag Phenomena in Gas-Filled Photoelectric Cells," Philips' Technical Review, 4:52, Feb., 1949

at the edges of the spot to be very slow. A reasonable assumption would be that if any increase in operating voltage is applied through the application of a variational voltage, the ionization would increase at the edges but so would the losses. It is for this reason that the spread of the glow is at a slow rate.

Reference to Figure 1 shows that the variational current at low glow currents increases rather rapidly with decreases in frequency. This bears out the conclusion that a process which is very slow is in operation. The process described in the preceding paragraph may be the mechanism.

If it is assumed that the variational current

$$i = f(x, t) \tag{13}$$

then

$$di = \frac{\partial i}{\partial x} dx + \frac{\partial i}{\partial t} dt \tag{14}$$

and

$$\frac{di}{dt} = \frac{\partial i}{\partial x} \frac{dx}{dt} + \frac{\partial i}{\partial t} \tag{15}$$

Experimental measurements will give some idea of the functions on the right of Equation 15. Figure 13 shows the rather remarkable relationship between variational current and voltage for a glow discharge. From this data it may be concluded that the spread of the glow at low frequencies is proportional to the magnitude of the variational voltage

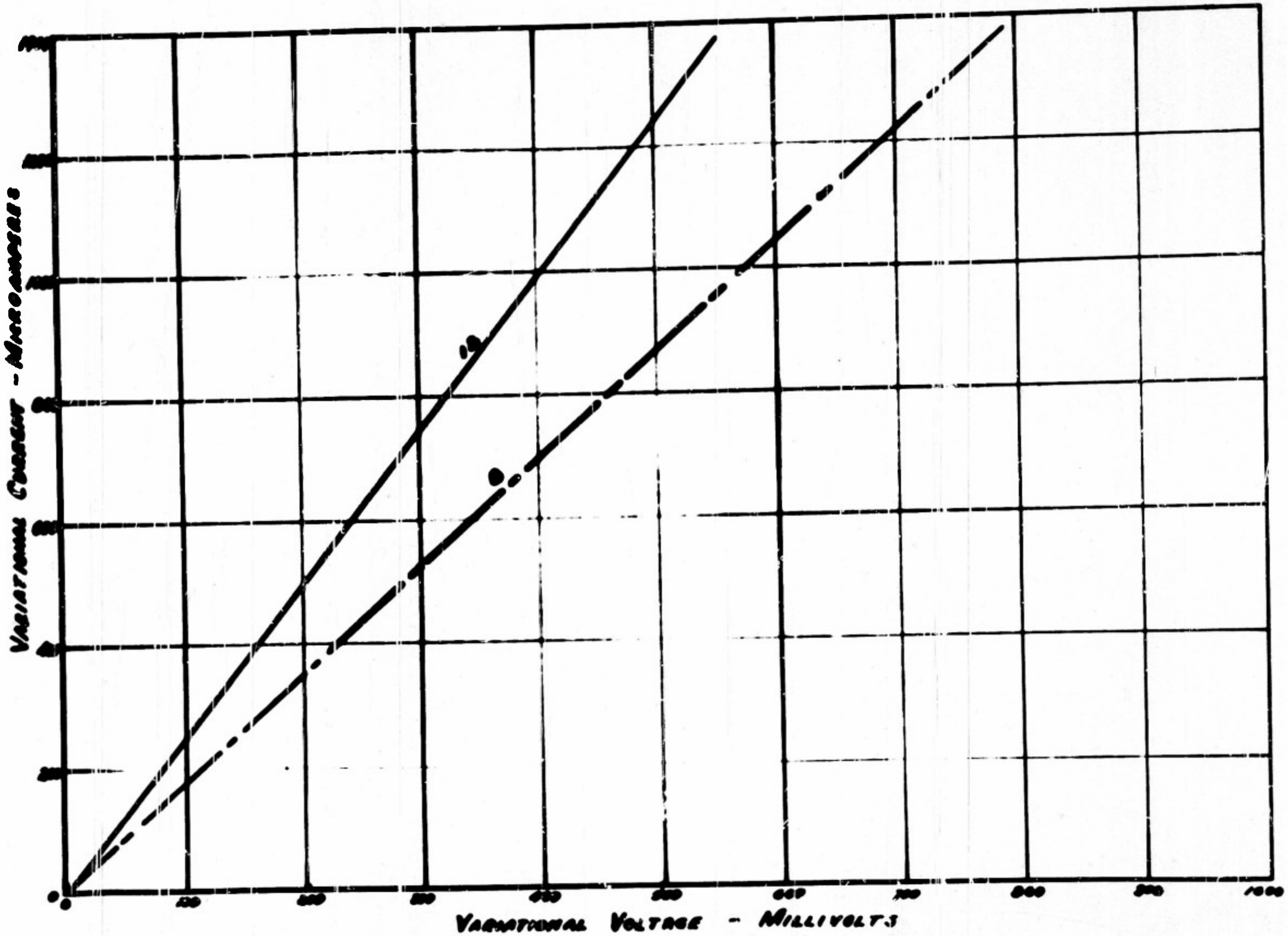

Fig. 13. The relation of variational current to variational voltage in a normal glow discharge at 500 cycles per second. Parameter-direct current glow current.

since it is reasonable to assume that the current density distribution as the glow spreads is the same. That is, the velocity of the spread of the glow

$$\frac{dx}{dt} = ke \tag{16}$$

where e is the variational voltage and k is a constant. It is assumed that the second term in Equation 15 is the derivative of the current described by Equation 5. This must be so since the second term represents the rate of change of variational current while the glow is not allowed to spread. This is the constrained discharge. The solution of Equation 15 would be of this form

$$i = \int \frac{\partial i}{\partial x} ke\,dt + \int \frac{\partial i}{\partial t}\,dt \tag{17}$$

It is assumed that the second integral is Equation 5. The solution cannot be completed because there is no information on the current density distribution of the glow at the edges and the first integration cannot be carried out. If it is assumed that the current density distribution at the edge of the spot is an exponentially decaying function, the first integration of Equation 17 can be carried out. The resulting solution while of the correct form is a very poor fit to the data.

XI. CONCLUSION

It is concluded, as a result of this study, that the time delay of variational current in reference to the applied variational voltage is the result of the build-up time in ionization. This is true at high

and low frequencies and at high and low D.C. glow currents.

The temperature of the cathode seems to have no major effect on the time delay.

Electrode configuration is not an important factor in the response curves of the variational currents.

Metastable atoms play an important role in the direct current glow discharge. Metastable atoms, however, play only a minor role in the variational discharge characteristics as is borne out by the data of Figures 6 and 7.

There seems to be no way to eliminate the large time delays inherent in glow discharge voltage regulator tube. If it were possible to eliminate these effects, the usefulness of the glow tube in electronic circuits as a circuit element would be greatly increased.